Five Year Integrated M.Sc. Examination, 2022-2023
Semester - V
Code: PH-3-5-5

Time: Three Hours | Subject: Statistical Mechanics | Full Marks: 40

Questions are of values as indicated in the margin
Answer question number **one** and any **three** from the rest

1. Answer **any five** questions

$5 \times 2 = 10$

(a) What is the probability of throwing a total of 5 points or less with three dice?

(b) "Quantum identical particles are indistinguishable"- Justify .

(c) Define reversible and irreversible processes in terms of change in number of accessible states $\Omega(E, V, N)$.

(d) What is the dimension of the phase space of a rigid body? Explain your answer.

(e) State and explain the postulate of equal a priori probability.

(f) Consider a system of N number of non-interaction particles distributed over three energy levels ϵ_1, ϵ_2, ϵ_3 with degeneracy g_1, g_2 and g_3 respectively. This system is in thermal equilibrium with a reservoir at temperature T . Write the canonical partition function for this system.

(g) Starting from the definition of microcanonical entropy for microcanonical ensemble, show that the entropy is an additive quantity.

2. (a) A particle performs random walk motion in one dimension about the origin O , and at each step it moves by a unit distance either to the left or to the right. Suppose the probability of its being to the right is p, while the probability of its being to the left is $q = 1 - p$. Show that after a total N steps the particle will be found at n steps right to the origin (O) with the probability distribution

$$W_N(n) = \frac{N!}{n!(N-n)!}\, p^n (1-p)^{N-n} \, .$$

(b) Consider the above mention probability distribution for a special case where the probability p is small ($p << 1$) and $n << N$. (i) Now using the result $\ln(1-p) \approx -p$, show that $(1-p)^{N-n} \approx e^{-Np}$, (ii) Show that $N!/(N-n)! \approx N^n$ and (iii) Hence show that $W_N(n)$ reduces to

$$W_N(n) = \frac{\lambda^n}{n!} e^{-\lambda} \, ,$$

where $\lambda \equiv Np$ is the mean number of events.

5+5=10

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3. (a) Consider an isolated system A^0 which is divided into two sub systems A and A'. Since $A^0 \equiv A + A'$ is isolated, its energy $E^0 = E + E'$ is constant. Sub systems A and A' can only exchange energy among themselves. Establish the condition for thermal equilibrium between A and A'.

   (b) Derive the partition function for canonical ensemble. Clearly state the assumptions that one needs to adopt to derive canonical partition function.

   (c) Stating from the canonical partition function show that the energy fluctuation (ΔE) is proportional the heat capacity C_V.

3+3+4=10

4. (a) Compute the canonical partition function, $Z_N(T)$ for an Ideal gas with N particles confined within a volume V in two dimensions (2D). Hence calculate the average energy of this system.

   (b) For a canonical ensemble described by the partition function Z, show that (i) average pressure , $\bar{p} = k_B T \frac{\partial \ln Z}{\partial V}$ and (ii) free energy, $F = -k_B T \ln Z$.

5+5=10

5. (a) Consider a micro-canonical ensemble of N number of bosons with total energy E. These bosons are distributed over large number of degenerate energy levels where i-th energy level having energy ϵ_i and degeneracy g_i contains n_i number of particle. Find a expression for the number of states for a particular configuration $\{n_i\}$.

   (b) Using the above expression, derive the most probable distribution for bosons.

   (c) Energy of the n-th eigenstate of an one dimensional quantum linear harmonic oscillator (LHO) is given by $E_n = (n + \frac{1}{2})\hbar\omega$. Calculate the canonical partition function of a system consisting of N number of such identical LHO with same frequency ω, which is in equilibrium with a thermal reservoir at temperature T.

3+4+3=10